Transitioning between the tableau and web bases

Jieru Zhu (joint with Mee Seong Im)

University at Buffalo

Irreducible S_n-modules: parametrized by partitions

Fix partition (n, n)

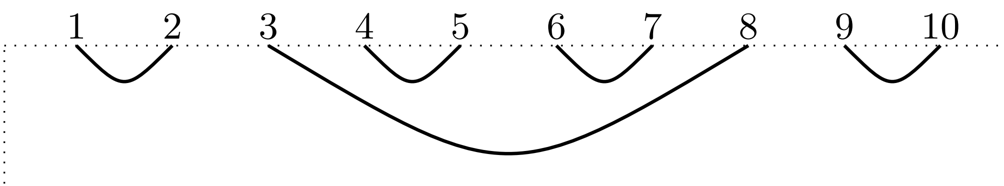
Standard Young Tableau (STY): a filling of (n, n) with $1, 2, \ldots, 2n$ which satisfies the standard condition

e.g.

1	2	4	5	7
3	6	8	9	10

Cup diagrams / A_1-webs: non-crossing arcs connecting $2n$ dots

e.g.



$S^{(n,n)}$: Specht module paramatrized by (n,n)

$$\begin{aligned} S^{(n,n)} &:= \operatorname{span}\langle v_T \mid T : \text{SYT of shape } (n,n)\rangle \\ &\simeq \operatorname{span}\langle W \mid W : \text{webs}\rangle \end{aligned}$$

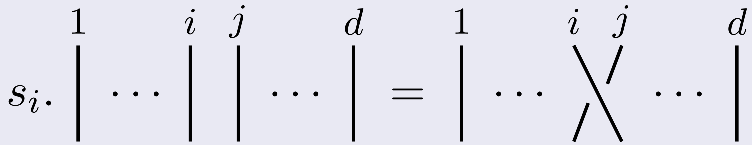
(a_{ST}): change-of-coordinate matrix from $\{v_T\}$ to $\{W\}$

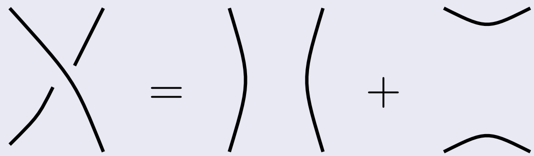
(Russell-Tymoczko, 2019)
(a_{ST}) is upper triangular

(Rhoades, 2019)
$a_{ST} \geq 0$.

(Im-Z.)
$a_{ST} > 0$ iff $S < T$.

Theorem (Im-Z.)





Theorem (Im-Z.)

$W = v_T$ for an explicit nonstandard T

$(a_{ST})^{-1} = (b_{ST})$

Conjecture: $b_{ST} = 0, \pm 1$

Question: When is $b_{ST} \neq 0$?