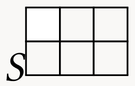
Transitioning between The Tableau Basis and The Spider Basis

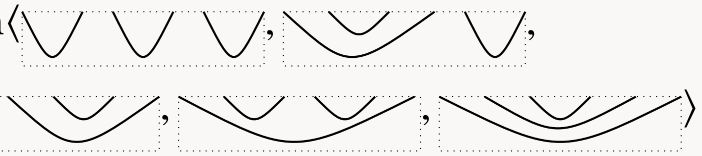
The Specht modules

Unique irreducible module for the symmetric group S_6 associated to the cycle type $(3,3)$:



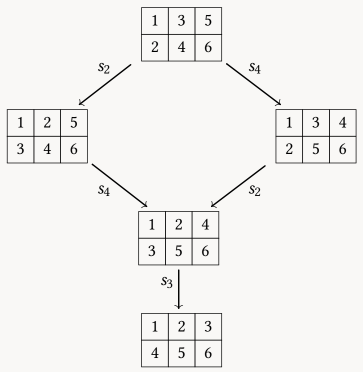
$:= \mathrm{span}\langle v_{\begin{smallmatrix}1&3&5\\2&4&6\end{smallmatrix}}, v_{\begin{smallmatrix}1&2&5\\3&4&6\end{smallmatrix}}, v_{\begin{smallmatrix}1&3&4\\2&5&6\end{smallmatrix}}, v_{\begin{smallmatrix}1&2&4\\3&5&6\end{smallmatrix}}, v_{\begin{smallmatrix}1&2&3\\4&5&6\end{smallmatrix}}\rangle$

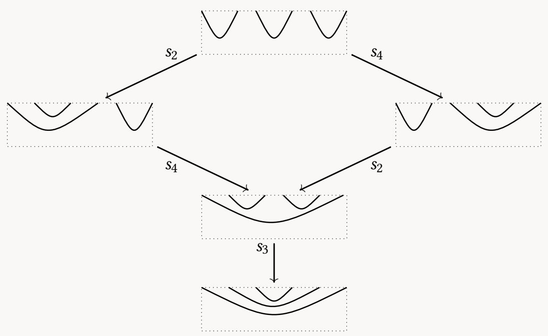
(via ϕ) $\simeq$ span⟨



The tableau graph v.s. the web graph

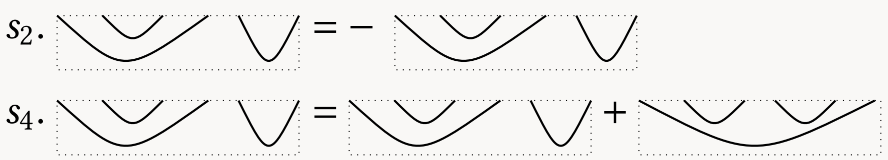
The two partially ordered sets are isomorphic:



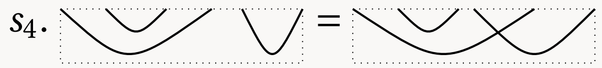


Action on a web

The S_6-module structure on webs:



Rephrasing the action



with new relation

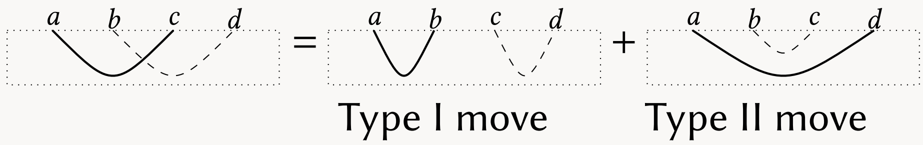
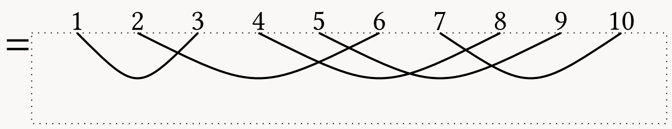
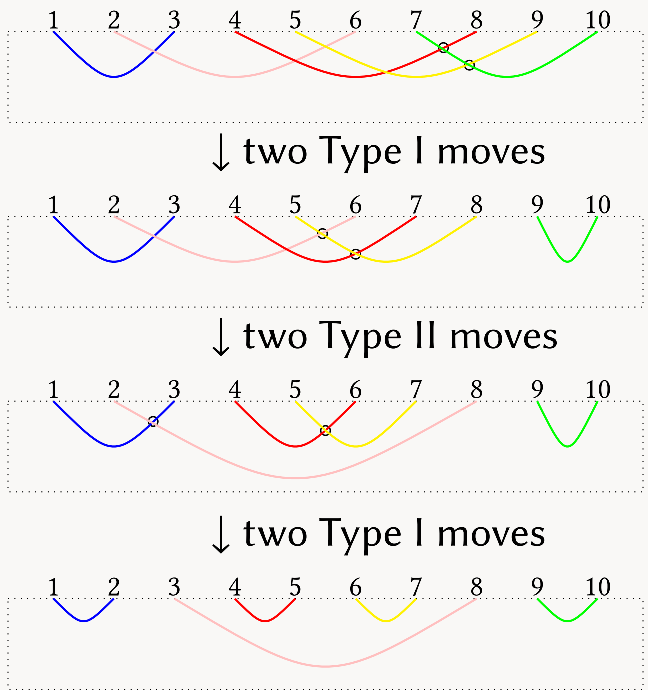


Image as a single web

$\phi\left(v_{\begin{smallmatrix}1&2&4&5&7\\3&6&8&9&10\end{smallmatrix}}\right) =$



Resolving the crossings: one example

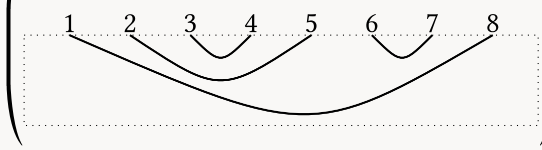


Theorem (Im-Z.)

The expansion of a polytabloid into webs, have positive coefficients.

Inverse of a web

$\phi^{-1}\Bigg($



$\Bigg)$

$= v_{\begin{smallmatrix}1&2&3&6\\8&5&4&7\end{smallmatrix}}$

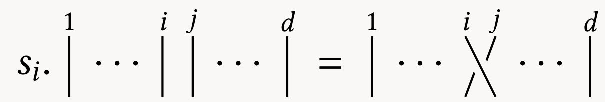
$= v_{\begin{smallmatrix}1&2&3&6\\4&5&7&8\end{smallmatrix}} - v_{\begin{smallmatrix}1&2&4&6\\3&5&7&8\end{smallmatrix}} - v_{\begin{smallmatrix}1&2&3&7\\4&5&6&8\end{smallmatrix}} + v_{\begin{smallmatrix}1&2&4&7\\3&5&6&8\end{smallmatrix}}$

A conjecture

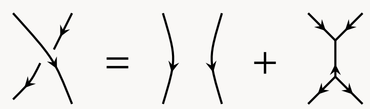
The expansion of a web into polytabloids has coefficients ± 1.

A peak into the spiders

Action:



Relation:



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