Pure Math Seminar

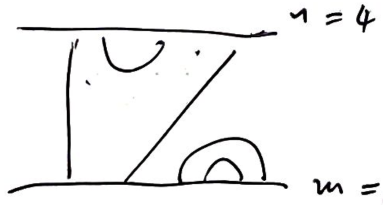
Temperley-Lieb algebra at roots of unity

jt w/ Sutton - Tubbenhauer - Wedrich
Benkart - Biswal - Kirkman - Nguyen

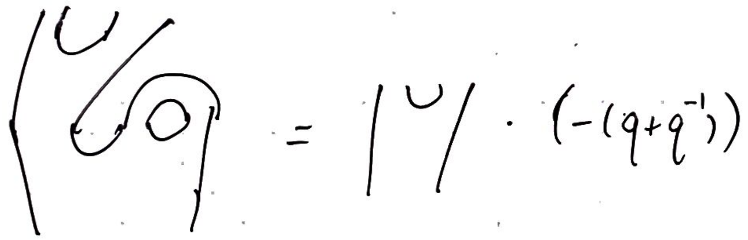
TL cat: a $\mathbb{k}(q)$-linear cat char $\mathbb{k}=0$ / char $\mathbb{k}=p$ q: generic or a root of unity

obj: $\mathbb{N}$

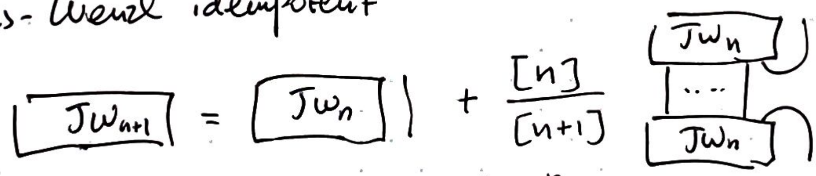
mor $\mathrm{Hom}(n,m)$ spanned by



composition: Stacking of diag

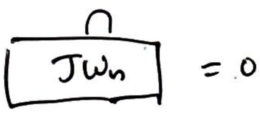


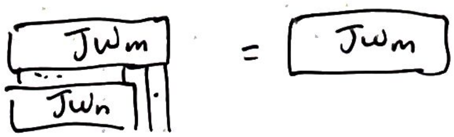
Jones-Wenzl idempotent



$$[n]=\frac{q^n-q^{-n}}{q-q^{-1}}$$

or: uniquely characterized by

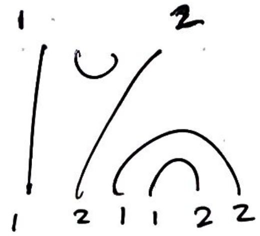
① 

② 

③ 

$\mathcal{F}$: $TL \longrightarrow$ sub-Cat of $U_q(\mathfrak{sl}_2)$-mod gen. by $\oplus, \otimes$ of $V = \mathbb{k}(q)^{\oplus 2}$

$n \longmapsto V^{\otimes n}$



$\in \mathrm{Hom}_{U_q(\mathfrak{sl}_2)}(V^{\otimes 6}, V^{\otimes 4})$

e.g. $f(v_1 \otimes v_2 \otimes v_1 \otimes v_1 \otimes v_2 \otimes v_2) = v_1 \otimes v_1 \otimes v_2 \otimes v_2$
$+ v_1 \otimes v_2 \otimes v_1 \otimes v_2$

Th (Rumer - Teller - Weyl, 1932
Cautis - Kamnitzer - Morrison, 2014 $U_q(\mathfrak{sl}_n)$)

$\mathcal{F}$ is full.

Karoubi envelope $Kar(TL)$:

obj (n, e)
e: idempotent in $\mathrm{Hom}(n, n)$

Morphism $(n, e) \to (m, e')$ is $f: n \to m$

$e' f e = f$

For $U_q(\mathfrak{sl}_2)$-mod $S^2(V) = V \otimes V / \sim$
$\sim$: $v \otimes w = w \otimes v$

$S^n(V) = \Delta(n) = T(n)$

$T(n) / \Delta(n)$: occurs as largest summand of $V^{\otimes n}$

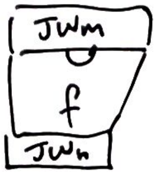
$T(n)$ closed under $\otimes$.

$V^{\otimes n} \longrightarrow T(n) \hookrightarrow V^{\otimes n}$ $\qquad JW_n$

($\Delta(n)$ in char 0)

$\mathcal{F}$: $(n, JW_n) \longmapsto \Delta(n)$

also: $f: (n, JW_n) \to (m, JW_m)$



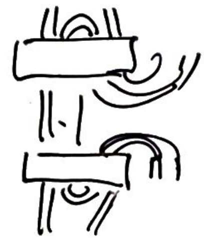
$= f \qquad f = 0$ for $m \neq n$

$\mathcal{F}$: $\mathrm{Kar}(TL) \simeq \mathrm{Tilt}$: the cat. gen. by $\otimes, \oplus$ of $T(n)$

(Burrell-Libedinsky-Sentinelli) char $\Bbbk = p$. $q = 1$

p-JW : $V^{\otimes n} \to T(n) \to V^{\otimes n}$

$pJW_n = \Sigma$



(Th) im pJW_n : is $T(n)$

$T(n)$: the module which admits a Δ-filtration and a ∇-filtration.

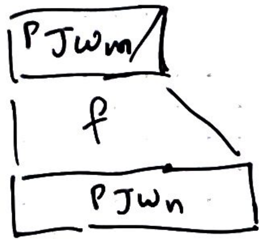
(Tubbenhauer-Wedrich)

(Sutton-Tubbenhauer-Wedrich-Z.)

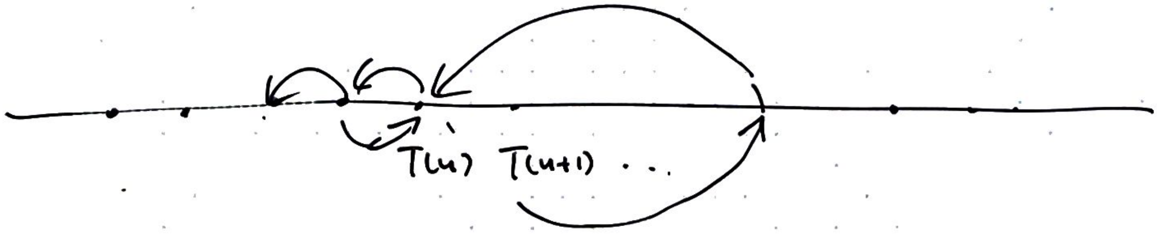
trace formula for p-JW.

/recursive

Th there exists at most one morphism between (n, pJW_n) and (m, pJW_m)



Quiver for Tilt:



& relations....

$D_n = T \otimes T^*$ alg over $\mathbb{k}(q)$: q a n^{th} primative root of unity

has module $V = V(2, \frac{n+1}{2})$ $V \triangleq V^*$

(Chen) D_n: quasitriangular Hopf alg w/ $R \in D_n \otimes D_n$

(Ram-Leduc) $Br_d \to \mathrm{End}_{D_n}(V^{\otimes d})$

$\times \mapsto R \curvearrowright V \otimes V$

(BBKNZ) $Br_d \to H_d \to TL_d \to \mathrm{End}_{D_n}(V^{\otimes d})$

(Goodman-Wenzl) $TL_d \to \mathrm{End}_{D_n}(V^{\otimes d})$ injective

(BBKNZ) surjective for $d \leq 2n-2$