

In the paper by Fukawa-Connelly et al., the authors examined a 10-minute class segment in which the teacher taught a specific proof, and claimed that there was a discrepancy between the main idea the teacher tried to convey and what the students actually perceived. They reached this conclusion by comparing their summary of the class with the student interviews conducted in three fashions. In addition, they conjectured that the reason causing such discrepancy is the omission of blackboard writings of the main point in the authors' view. The reasons they gave supporting this conjecture is that most students only took blackboard notes, and that the students could only recall the key point immediately after watching it in videos.

In the paper by Simon, he based his research on the constructivists' view on the human learning mechanism, i.e. learning is the process of adapting one's perception of the experiential world when there is a disequilibrium between what's perceived and what's expected. The writer didn't choose a specific school between radical constructivist and social constructivist but took a combination of both. He also claimed that this assumption on learning mechanism doesn't prescribe a specific way of teaching, and proposed his own "theoretical framework" as a result of applying these views. In this framework, he thinks there is a symbiotic relationship between teacher's planning and his/her hypothesis about students' learning trajectory: the teacher should try to anticipate a learning trajectory based on his past experience and guesses, and use this to form his teaching goals and plan class activities. After observing students' responses in class, s/he should assess their understanding and base further expectations on this assessment, therefore form further hypothetical learning trajectory.

My own thought after reading the two paper deviates away from the two papers' original intentions. In the paper by Fukawa-Connelly et al, the students kept mentioning the phrase "bunching up", and I think this can be explained by the constructivists' view in the sense that their intuition led them expect that "bunching up" should be the key characteristic of a convergent sequence. In the paper by Simon, it is interesting that the author thinks the teacher should anticipate their learning trajectory as a progressing path of thought development, instead of a transient state of mathematical mind. It is also impressive that he mentioned the the teacher has no access to the actual learning trajectory of the students, and even the assessment about their understanding is hypothetical. My own opinion is that it is helpful to compare the two trajectories, and pose the question of what caused the discrepancy: the author seems to believe that this discrepancy is mainly caused by the unpredictable nature of social interactions in class, whereas I think other reasons may be more important, such as the teacher's unawareness of the students' certain thinking patterns. It would also be interesting to find tangible ways to measure the difference between the hypothetical and "actual" learning trajectory, for example in terms of specific ways students phrase their answers.