

In the paper by Sztajn et al., the authors introduced the framework of LTBI (learning trajectory based instruction) based on the previous focus on the (LT) learning trajectory originated by scholars such as Simon. This framework stresses on the importance of utilizing research based learning trajectory relevant to specific topics, and past experiments have shown that it has certain benefits in teaching. They referenced earlier theoretical frameworks that were developed from the original constructivists' point of view, and gave the LTBI interpretations for each of the frameworks in four categories: mathematical knowledge for teaching, task analysis, pedagogical practices for orchestrating discourse, and formative assessment. Under this new framework, they were able to incorporate the earlier theories into one entity.

In the paper by Doorman, he introduced the theory of RME (realistic mathematics education) proposed by past scholars such as Kaput and Freudenthal, in which students are exposed to context problems that are relatable to their everyday life, so that they can develop their own models to be later transitioned to a mathematical setting. Under such guidelines the emergent models are used by teachers when designing their class tasks, to let the students reinvent mathematics in a guided way, and Doorman discussed an example where the students learned the unit measure on the real line. He then gave a historical overview of the "emergent models" appeared in the invention of kinematics, where mathematicians find their models for the relations between distance and time in the process of defining derivatives. In the end, he discussed a case where he used the RME in a calculus class using the historical emergent models as a guideline.

In the paper by Larsen, he introduced his theory of LIT (local instructional theory) based on the RME view held by scholars such as Freudenthal. He conducted informal studies in various settings, from choosing pairs as subjects, teaching a bridge course, to finally conducting the study in a college abstract algebra course in collaboration with mathematicians as teachers. He used the philosophy in RME and observe the transition among students from "model-of" to "model for" a mathematical concept. The subject matter is the definition of a group and the concept of group homomorphism. He first created a context problem where one examines the set of symmetries (transformation operations) on an equilateral triangle. The students are guided to choose symbols and derive tables to calculate the composition as the group operation. From the table they were able to find a set of efficient and complete rules to axiomatize the definition of a group. Later, by giving the students a "mystery group", the author managed to guide them into reinventing the concept of group homomorphism in terms of an algebraic formula.

Despite my reserved opinions about LTBI and RME, what interests me in Doorman's paper is that he used the historical development of calculus as a guideline for

designing context problems. I have learned a few stories about different classes where material was presented in the way that followed the historical development pattern. In particular, in one of the calculus classes that I TAed for, the professor spent the first class discussing the history of mathematics up to the point of calculus, and the subject matter became very motivated. This would be a nice point to be kept in mind in my future teaching.