In the paper by Moore, the author begun by mentioning the education atmosphere in which most students encounter an abrupt transaction to proof centered courses. By examining several transaction-to-proof courses at a university, the author observed seven major difficulties(D1-D7) that students may have encountered in such courses. After combining them into a diagram explaining the interactions, he proposed a theory of concept-understanding scheme to explain the cause of such difficulties. The author used experiment data to illustrate each of the three parts in the scheme. The concept definition part requires the students to use rigorous mathematical language, and provides more insights into the logic structure of proofs, but are difficult for the students to master. This results in students' difficulty in recalling a definition(D1). The concept image which gives students an intuition to understand the definition(D2), help students to recall the definitions effectively, but sometimes gives the students a sense of understanding the concept without being able to use mathematical language, and an uncertainty about how to start a problem(D3). The concept usage is the students' ability to apply the definition in different contexts, and the author further divided into categories including the ability to generate examples(D4) and structure proofs using definitions(D5). As for recommendations for teaching, the author suggests a image-definition-usage sequence, and proposed the reinforcement of transaction-to-proof courses in college education.

In the paper by Boyle and Byrne, the authors proposed a qualitative assessment tool to evaluate mathematics arguments. They first commented on the current undergraduate mathematics education that students don't usually know what count as a proof and their tendency to memorize proofs, to illustrate the difficulty of teaching proofs in a constructivist classroom without feedback tools. They further developed an assessment tool based on previous results and categorized arguments as A1,A2,A3,A4, with A4 being a satisfactory proof and subdivisions at each category representing the degree to which it resembles a proof. They conducted two studies in which they used the tool to assess student arguments and were able to achieve consensus on the grading. They further gave two examples of students' arguments using such a tool and proposed future study, where the assessment tool could be used as instructional tools to provide feedback to students.

In the paper by Bloom and Carlson, the authors proposed an "emergent multidimensional problem-solving" framework. They first referenced scholars such as Polya, Garofalo and Lester, to propose different aspects and stages, individual for each problem solvers, of the problem solving process. Based on these aspects they posed their first attempt at a taxonomy of coding patterns appeared in the process. They then conducted the study by examining 11 mathematicians solving four problems that fit their definition of a good mathematics problem. By videotaping and taking the transcript of the data, they coded the data with their first taxonomy. During the analysis they realized the limitation of such taxonomy in the lack of description of interactions within different stages of thinking, after which they proposed their "emerging multidimensional problem-solving" framework. In their framework, they claimed that experienced problem solvers go through stages such as orienting, planning, executing and checking, similar to what Polya has described. In the planning stage, they also experienced a conjecture cycle consisting of conjecture, imagine and evaluate. In each of the stages, problem solvers display "multidimensional" aspects in their thinking, which represents the four dimensions mentioned in the first taxonomy, also similar to opinions by scholars such as Garofalo, Lester, Schoenfeld, DeBellis and Goldin. Such dimensions included resources, one's mathematics knowledge and ability to apply them; hereustics, the strategic approaches one can take; affect, one's emotions regarding mathematics and the ability to control them; and monitoring, regularly checking one's result for conviction. To explain the framework, they referenced the transcripts of the video tapes along with their interpretations using the coding taxonomy. They observed the dimensions used in different stages that are represented by behaviors distinct to that stage, and concluded a table representing such specifications.

What stood out to me the most in Moore's paper, is the recording of the interview of a student proving the set theory problem about the intersection and union of two sets. The student's response is actually wrong in logic, because she confused the original statement with the converse, a classical mistake students would make. I think this is an excellent opportunity to ask her why she would draw that Venn diagram, and why the diagram shouldn't be otherwise. As soon as she starts defending her diagram, she would start making a logical argument which is the heart of the problem. I think sometimes students feel intimidated by logic formality because they view logic as a set of authoritative rules they have to obey, without realizing them as ways to establish the common sense of reasoning. This common sense would be established by the back-and-forth debate between the teacher and the students, before they are able to appreciate the formality. On the other hand, Boyle 's paper gave a more objective assessment of students' proof construction. Since I had been dissatisfied by the lack of objective assessment in some papers regarding students' achievement, I find this to be an excellent solution. For example, when researching the effectiveness of certain methods in a transaction-to-proof course, we could convert each of the letter grade A1,A2,A3,A4 to a percentage, and calculate the average of student response in both the experiment and control groups. The paper by Bloom and Carlson is a very systematically written paper, quoting very detailed data and a good review of previous results. As a PHD student I have experienced the cycle they mentioned both in my courses and my project, and I agree with their mention of the "monitoring/control" in which one regularly checks one's result. They mentioned in their paper that this was an important quality of experienced problem solver, as I have also witnessed the lack of it when I tutor students. It will be interesting to explore the question of what contributes to this aspect in some problem solvers and the lack of it in others.