In the paper by Dubinsky and McDonald, the authors proposed and discussed the various aspects of the "APOS" theory. They begun their discussion by mentioning six features that a mathematics education research theory should possess and proposed the "APOS" theory under such standards. Based on Piaget's theory on Children's learning, they proposed four components of the students learning and used the example of cosets in groups to explain students learning in the light of such theory. Students would take explicit "Actions" or operations on a specific example, and have the "Process" of reflecting on the actions without doing them in detail. Afterwards they would view the processs and actions as an "Object" in its totally where transformations could act on them, and form a "Schema" consisting of their understanding of the concept or its applications. To discuss the value of such theory with respect to research, they further suggested a study based on this theory, in which students are instructed to use computer language with mathematical syntax to write programs computing elements in the cosets. Further exercises and evaluations could be performed afterwards to observe the results. They also mentioned possible refinement of such theory using the triad theory by Piaget and Garcia, its reception and effect in the RUMEC society, and referenced a list of bibliographies of researchers who have applied such theory in their studies.

In the paper by Vinner and Tall, the authors defined the term "concept definition" as the form of words used to specify the concepts, either personally by the students or formally in the mathematical society, and "concept image" as cognitive processes by which they are conceived. They further illustrated that one can develop several concept images when encountering the same concept at different situations, and concept images can conflict with each other as well as the conflict definitions do. They explored the different "potential conflict factors" that may contribute to such conflicts, in the setting of a program using The School Mathematics Project Advanced Level texts(SMP) that are designed to carefully cultivate different concept images in a spiral manner before reaching a concept definition. By giving students specific problems and theorems on three concepts, they were able to see students' response to several potential conflict factors. In particular, for the limit of a sequence, students would fail to identify $0.\dot{9}$ as a limit problem; they also encountered conflicts when sequences are not given by a single formula. On the limit of a function, students would have a dynamic understanding of the limit while failing to include $x \neq a$ in the definition of $\lim_{x \to a} f(x)$. On the continuity of a function, some students would have the concept image of continuity as functions being given by a single formula, or see it as a global property instead of being at a given point when it comes to the Dirichlet function.

What I appreciate in the APOS theory is its focus on the "hands-on" experience,

i.e. their "action" part in the APOS. This reminds me of the empirical evidence in the paper by Weber et al., and I totally agree that mathematicians and students alike, gain a lot of understanding through working out actual examples. As we give a lecture, we should take every advantage to let the students work out the details themselves, instead of handing them the result, since I notice that students get bored easily if not given an opportunity to participate mentally. From the paper by Vinner and Tall, I agree that we as mathematicians also have both informal and formal understanding of mathematical concepts. The professors I have had great experiences with are usually very skillful at cultivating the informal concept image as well as capturing the image with formal concept definition. In addition to the authors' opinion that the concepts should be introduced in such an order that cultivates building of the correct concept image, I also think students should be prompted to write formal definitions in their own language, and then given feedback from the teacher why their way of defining is effective or ineffective. I think the way teachers give feedback would influence in a great way, how students conceive the process of converting concept image to concept definition.